Tutorial 10.

Preliminary:

① Dollar-weighted return:

A: balance in a fund at the start of the year;

B: balance in the fund at the end of the year;

C_k: net deposit at time t_k.

$A(1+i)+\sum_{k=1}^{n} C_k(1+i)^{1-t_k}$ is accumulate value. ~~is this on t_k~~. $\qquad (1+i)^{(1-t)} \approx 1+(1-t)i.$

$$A(1+i)+C_1(1+(1-t_1)i)+\cdots+C_n(1+(1-t_n)i)=B$$

$$\left(A+\sum_{k=1}^{n} C_k\right)+i\left(A+\sum_{k=1}^{n} C_k(1-t_k)\right)=B$$

$$i=\frac{B-\left(A+\sum_{k=1}^{n} C_k\right)}{A+\sum_{k=1}^{n} C_k(1-t_k)}.$$

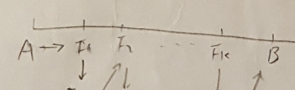
$I=B-\left(A+\sum_{k=1}^{n} C_k\right)$ is the net amount of interest.

② Time-weighted return:

F_k: the value of fund just before the net deposit C_k at time t_k,

$$i=\frac{F_1}{A}\times\frac{F_2}{F_1+C_1}\times\cdots\times\frac{F_k}{F_{k-1}+C_{k-1}}\times\frac{B}{F_k+C_k}-1$$

$\frac{F_j}{F_{j-1}+C_{j-1}}$ is growth factor from t_{j-1} to t_j.



return in time t_k $\Rightarrow$ $j_1=\frac{F_1}{A}-1,\ j_2=\frac{F_2}{F_1+C_1}-1,\ \cdots\ j_{k+1}=\frac{B}{F_k+C_k},$

$(1+j_1)\cdots(1+j_{k+1})=1+i.$

③ Yield measure on a fund

$$i \doteq \frac{2I}{2\int_{t_1}^{t_2} F(t)\,dt-I}=\frac{2I}{F(t_1)+F(t_2)-I}.$$

the amount in the fund at time t is $F(t)$.

Exercise:

5-2-3.

	Value	withdrawal
Jan 1, 2013	100,000	
April 1, 2013	103,000	-8000
Jan 1, 2015	103,992	

x is annual rate

the balance at the end of 2013:

$100{,}000(1+x)-8000(1+\frac{3}{4}x)$

at the end of 2014

$(100{,}000(1+x)-8000(1+\frac{3}{4}x))(1+x)=103{,}992.$

$\Rightarrow x=0.0625$

5.2.6.

dollar weighted return for K,

$$100(1+i) - X(1+\tfrac{1}{2}i) + 2X(1+\tfrac{1}{4}i) = 125$$

$$\Rightarrow X = 125 - 100(1+i).$$

time weighted return for L.

$$\frac{125}{100} \times \frac{105.8}{125-X} = 1+i \Rightarrow X = 125 - \frac{132.25}{1+i}$$

then $125 - \frac{132.25}{1+i} = 125 - 100(1+i) \Rightarrow i = 0.15.$

5.3.1.

(a) $F(0) = 500$, $F(1) = 500 + 100 - 40 + 60 = 620$. $I = 60$,

yield rate $i = \frac{2I}{F(0)+F(1)-I} = \frac{120}{500+620-60} = 0.1132$

(b) (i) $F(0) = 500+100 = 600$, $F(1) = 620$, $I = 60$.

$$i = \frac{120}{600+620-60} = 0.1034.$$

(ii) $F(t) = \begin{cases} 500+20t & 0 \le t \le \frac{1}{4} \\ 600+20t & \frac{1}{4} \le t \le 1. \end{cases} \Rightarrow \int_0^1 F(t)dt = \int_0^{\frac{1}{4}} F(t)dt + \int_{\frac{1}{4}}^1 F(t)dt = 585.$

$$i = \frac{120}{2\times 585-60} = 0.1081.$$

$F(t) = \begin{cases} 500+20t & 0 \le t \le \frac{1}{2} \\ 600+20t & \frac{1}{2} \le t \le 1. \end{cases}$

(iii) $\int_0^1 F(t)dt = \int_0^{\frac{1}{2}} F(t)dt + \int_{\frac{1}{2}}^1 F(t)dt = 560 \Rightarrow i = \frac{120}{2\times 560-60} = 0.1132$

$F(t) = \begin{cases} 500+20t & [0,\frac{3}{4}) \\ 600+20t & [\frac{3}{4},1] \end{cases}$

(iv) $\int_0^1 F(t)dt = \int_0^{\frac{3}{4}} F(t)dt + \int_{\frac{3}{4}}^1 F(t)dt = 535 \Rightarrow i = \frac{120}{2\times 535-60} = 0.1188$

$F(t) = 500+20t$

(v) $\int_0^1 F(t)dt = \int_0^1 500+20t\,dt = 500 + 10t^2\Big|_0^1 = 510 \Rightarrow i = \frac{120}{2\times 510-60} = 0.125.$